MODE COUPLING AND POWER TRANSFER IN A COAXIAL SECTOR WAVEGUIDE WITH A SECTOR ANGLE TAPER

by

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ABSTRACT

We report a theoretical study of mode coupling and power transfer in a coaxial sector taper. The power transferred from the TE_{01} mode into other low lying modes is calculated as a function of taper length and operating frequency. This type waveguide taper is utilized to feed a wide band input coupler for a 35 GHz Gyrotron Travelling Wave Amplifier.

INTRODUCTION

The development of high power millimeter wave gyrotrons requires the availability of efficient over-moded waveguide tapers and couplers with high mode selectivity. Design criteria for tapers having circular or rectangular cross section have been discussed, but there have been few analyses of more complicated geometries. In this paper we report results from a theoretical study of mode propagation and coupling in a taper consisting of a sector of a coaxial waveguide with a variable sector angle. A practical application of this taper is the transformation of the TE_{10} mode in a rectangular waveguide into the TE_{01} mode of a coaxial waveguide. This approach has been utilized for matching a rectangular waveguide to a broad-band coaxial input coupler in a Gyro-TWT amplifier [1].

Our analysis of mode coupling in the taper is based on an approximate form of the generalized telegraphist's equation for a nonuniform waveguide derived by Solymar [2]. We calculate the power transferred from the TE_{01} coaxial sector mode into other low lying modes. The dependence of the power transfer on taper length and on frequency is shown. An experimental program to verify these results is currently underway.

THEORY

If the electromagnetic fields in a nonuniform waveguide are expanded in terms of a set of transverse modes, these modes form a system of coupled transmission lines in which the field intensities in the waveguide correspond to equivalent voltages and currents. Neglecting losses, the generalized telegraphist's equation for a system of coupled transmission lines is of the form [2].

$$-\frac{dV_i}{dz} = j\beta_i K_i I_k - \sum_p T_{pi} V_p$$
$$-\frac{dI_i}{dz} = \frac{j\beta_i V_i}{K_i} + \sum_p T_{ip} I_p \qquad (1)$$

where V_i and I_i are the equivalent voltages and currents for the mode i, β_i is the propagation coefficient, and K_i is the wave impedance. The coupling of the transverse modes is characterized by the voltage and current transfer coefficients which are given by

$$T_{ip} = \int_{S(z)} \vec{e}_i \cdot \frac{\partial \vec{e}_p}{\partial z}\, ds \qquad (2)$$

where $\vec{e}_i$ and $\vec{e}_p$ are the mode vector functions and S(z) is the area of the waveguide cross section as indicated in Figure 1a. Expressions for the transfer coefficients in terms of line integrals over the boundary C(z) of the surface S(z) have been derived by Solymar [2].

FIGURE 1a

PARAMETERS USED FOR DESCRIPTION OF NONUNIFORM WAVEGUIDES

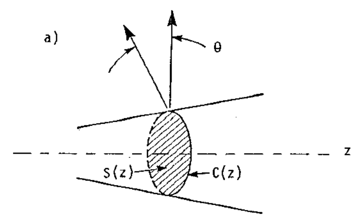
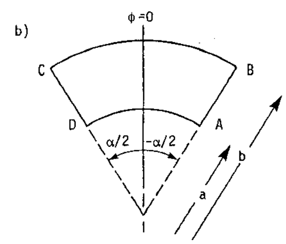


FIGURE 1b

COAXIAL SECTOR WAVEGUIDE CROSS-SECTION



We consider the coaxial sector taper which is symmetric about $\phi=0$ as shown in Figure 1b with sector angle α, inner radius a, and outer radius b.

It is convenient to transform the generalized telegraphist's equation to a representation in terms of forward and backward travelling waves. Introducing as new variables the amplitudes for forward and backward travelling waves:

$$A_i^+ = \tfrac{1}{2}\left(K_i^{-\frac{1}{2}} V_i + K_i^{\frac{1}{2}} I_i\right) \qquad (3a)$$

and

$$A_i^- = \tfrac{1}{2}\left(K_i^{-\frac{1}{2}} V_i - K_i^{\frac{1}{2}} I_i\right), \qquad (3b)$$

Eq. (1) becomes

$$\frac{dA_i^+}{dz} = -j\beta_i A_i^+ - \tfrac{1}{2}\frac{d(\ln K_i)}{dz} A_i^- + \sum_p \left(S_{ip}^+ A_p^+ + S_{ip}^- A_p^-\right)$$
$$\frac{dA_i^-}{dz} = +j\beta_i A_i^- - \tfrac{1}{2}\frac{d(\ln K_i)}{dz} A_i^+ + \sum_p \left(S_{ip}^- A_p^+ + S_{ip}^+ A_p^-\right) \qquad (4)$$

where S_{ip}^+, S_{ip}^- are the forward and backward coupling coefficients:

$$S_{ip}^{\pm} = \tfrac{1}{2}\left[\frac{K_p^{\frac{1}{2}}}{K_i^{\frac{1}{2}}} T_{pi} \mp \frac{K_i^{\frac{1}{2}}}{K_p^{\frac{1}{2}}} T_{ip}\right] \qquad (5)$$

For tapers of practical interest the power transferred into spurious modes is usually small compared to the power in the main mode and we may assume that the main mode is unaffected by the presence of other modes. If we further assume that other modes are excited only by the main mode m and that the coupled modes are above cutoff everywhere in the taper, the amplitude of the forward travelling wave in the mode i at the end of a taper of length L is given by [2].

$$A_i^+(L) = \exp\left[-j\int_0^L \beta_i dz\right] \int_0^L S_{im}^+ \exp\left[-j\int_0^z (\beta_m-\beta_i)dz'\right] dz \qquad (6a)$$

The amplitude of the backward travelling component of the main mode due to reflection is given by

$$A_m^-(0) = -\exp\left[-j\int_0^L \beta_m dz\right] \int_0^L \left(S_{mm}^- - \tfrac{1}{2}\frac{d(\ln K_m)}{dz}\right) \exp\left[-j2\int_0^z \beta_m dz'\right] dz \qquad (6b)$$

The relative power transferred into a spurious mode is given by

$$\frac{P_i^+}{P_m^+} = |A_i^+(L)|^2, \qquad (7a)$$

for a forward travelling mode, and by

$$\frac{P_m^-}{P_m^+} = |A_m^-(0)|^2 \qquad (7b)$$

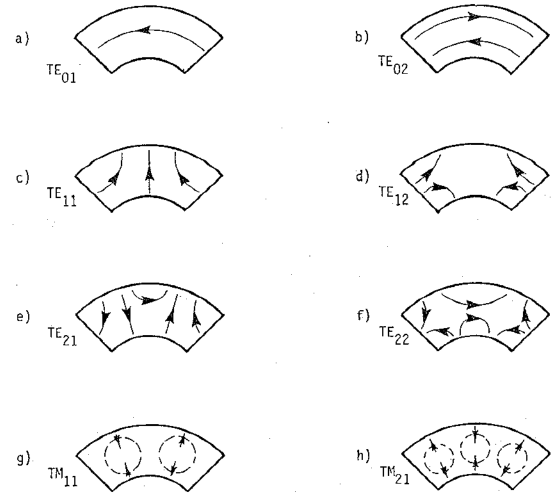
for the reflection of the main mode.

CALCULATIONS AND RESULTS

Schematic cross-sectional views of several low order coaxial sector modes are shown in Figure 2. The first mode subscript denotes the ϕ-index and the second denotes the r-index. If the taper is symmetric about the $\phi=0$ plane as in Figure 1b, and the inner and outer radii are held constant, the TE_{01} mode (Figure 2a) couples only to modes with even order ϕ-index. Thus the TE_{01} mode does not couple to the modes shown in Figures 2(c), (d), and (g). The coupling also vanishes between the TE_{01} mode and the TE_{02} shown in Figure 2b.

FIGURE 2

SCHEMATIC CROSS-SECTIONAL VIEWS OF LOW ORDER COAXIAL SECTOR MODES.
SOLID LINES INDICATE E FIELDS, DASHED LINES INDICATE H FIELDS SUBSCRIPT ORDER IS i_θ, i_r.



The cutoff frequency of a mode is related to its eigenvalue according to

$$\omega_c = \gamma_{mn} c \qquad (8)$$

where c is the speed of light. The cutoff frequencies of several low lying modes are shown as a function of sector angle in Figure 3 for a = 0.714 cm and b = 1.264 cm. If the inner and outer radii are held constant the taper cross section gradient angle θ is non-zero only along the radial line segments $\overline{AB}$ and $\overline{CD}$ in Figure 1b and varies with radius according to

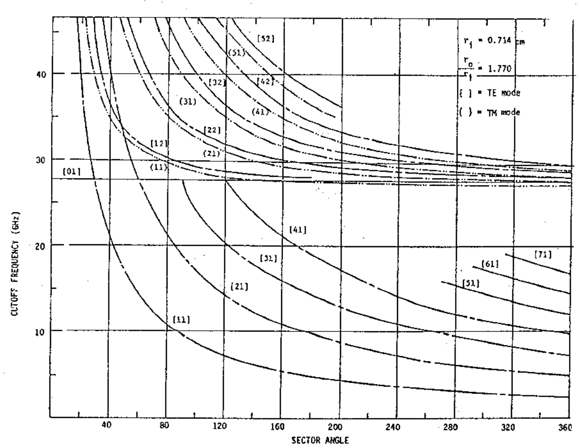
$$\tan\theta = \frac{d\alpha}{dz} r \qquad (9)$$

where $\frac{d\alpha}{dz}$ is the sector angle gradient. It is convenient to work with the normalized transfer coefficient

$$T' = T / \frac{d\alpha}{dz} \qquad (10)$$

FIGURE 3

CUTOFF FREQUENCIES OF LOW-LYING COAXIAL SECTOR MODES



Normalized transfer coefficients for coupling of the TE_{01} mode to the lowest even order modes are shown in Figure 4 as a function of sector angle. Note that the wave impedance of the TE_{01} mode

$$k_{[01]} = \frac{w\mu}{\beta_{[01]}} \qquad (11)$$

is independent of sector angle and, hence, z so that reflection of this wave occurs only via backward coupling and not by a change in wave impedance.

FIGURE 4

TRANSFER COEFFICIENTS FOR COUPLING OF TE_{01} MODE TO OTHER MODES

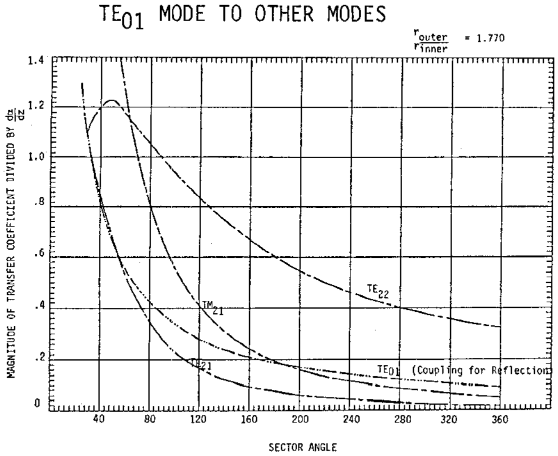


Figure 3 shows that at small sector angles only the TE_{01} mode can propagate (for frequencies above 27.62 GHz) since the cutoff frequencies of modes with non-zero ϕ-index vary approximately inversely proportional to sector angle in this region. The number of modes which can propagate increases rapidly as the sector angle is increased. To apply equations (6) and (7) for the power transferred into spurious modes we assume that the power transferred to a mode in the region of the taper where the mode is cutoff and, hence, evanescent is small and can be neglected. This should be a good approximation except at frequencies approaching cutoff of the TE_{01} mode. Figure 5 shows the power transferred into spurious modes from the TE_{01} mode as a function of the length of a linear taper (constant sector angle gradient) at the frequency 35 GHz. Figure 5 shows that the reflected power in the TE mode is much lower than the power transferred into forward travelling spurious modes at this frequency.

FIGURE 5

LENGTH DEPENDENCE OF POWER TRANSFER INTO SPURIOUS MODES IN A LINEAR COAXIAL SECTOR TAPER

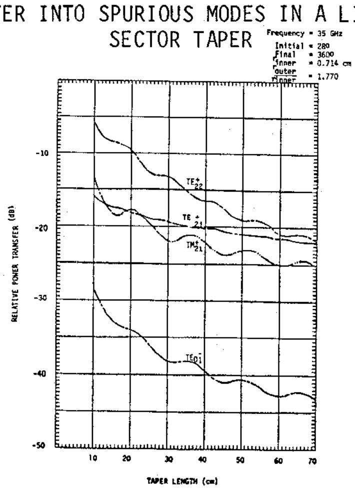
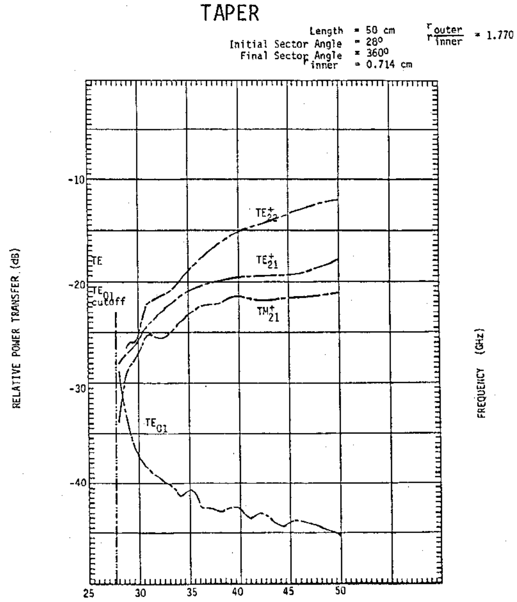


Figure 6 shows the frequency dependence of the power transfer for a linear taper of length 50 cm. The power transfer into the backward travelling TE_{01} mode increases rapidly as the frequency approaches cutoff of this mode. The results shown in Figure 6 may be in error at low frequencies due to the neglect of power transfer into the evanescent portion of the excited modes. In addition, coupling between the TE_{22}^{+} and TM_{21}^{+} modes, which has been neglected, may alter the relative power transferred into these modes. An experimental program is currently underway to test these results.

FIGURE 6

FREQUENCY DEPENDENCE OF POWER TRANSFERRED INTO SPURIOUS MODES IN A LINEAR COAXIAL SECTOR TAPER



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REFERENCES

[1] L. R. Barnett, M. Baird, A. W. Fliflet, and V. L. Granatstein, "Circular-Electric Mode Waveguide Couplers and Junctions for Use in Gyrotron Travelling Wave Amplifier," IEEE 1980 International Microwave Symposium Digest

[2] L. Solymar, "Spurious Mode Generation in Nonuniform Waveguide," IRE trans. MTT, p. 379, July 1959.

[3] G. Reiter, "Generalized Telegraphist's Equation for Waveguides of Varying Cross-Sections," presented at the Convention of Long Distance Transmission by Waveguides, London, England: 1959; Jan 29-30.

[4] N. Marcuritz, "Waveguide Handbook," McGraw-Hill Book Co., Inc., New York, N.Y. 1951.